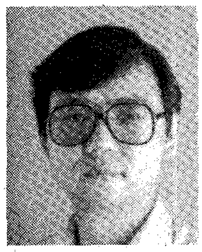

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Correction Due to a Finite Permittivity for a Ring Resonator in Free Space

RONALD DE SMEDT

Abstract —**To better determine the resonant fields of a dielectric resonator with high permittivity ϵ_r, the asymptotic theory with $1/\sqrt{\epsilon_r}$ as a small parameter is extended by adding higher order terms in $1/\sqrt{\epsilon_r}$ in the fields, the resonant wavenumber, and radiation Q. Extensive data are shown for the ϕ-independent "nonconfined" mode of a ring resonator, which radiates as a magnetic dipole. Some results are added for the "magnetic quadrupole" mode.**

I. Introduction

THE CHARACTERISTICS of a dielectric resonator of high permittivity, an important component of microwave circuits [1]–[2], have been investigated extensively [3]–[12]. A solution for arbitrary ϵ_r requires the solution of the field problem for each ϵ_r encountered in practice. This cumbersome procedure can be avoided by introducing a perturbational approach based on a series expansion in $1/N = 1/\sqrt{\epsilon_r}$ [9]–[11]. The leading term in these series gives good results as soon as ϵ_r exceeds, say, 100 [11], [13], [14]. Present resonators, however, are based on materials with ϵ_r of the order of 38, because these materials have better temperature coefficients, lower losses, and are more reproducible. The present paper endeavors to extend the limit of applicability of the perturbational approach by evaluating higher order terms in the series in $1/N$. Numerical data are given for the lowest ϕ-independent "nonconfined" resonant modes of a ring resonator located in free space (Fig. 1). These modes radiate either as a magnetic dipole or a quadrupole. The quadrupole mode satisfies an "electric wall" condition in the $z = 0$ plane and, hence, is relevant for a resonator located on a metallic plane [13].

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II. Mathematical Formulation

A. Fields

The ϕ-independent fields with azimuthal $\bar{E}$ of an axially symmetric resonator can be derived from a scalar function $\alpha(r, z)$, according to

$$\begin{cases} \bar{E} = -j\dfrac{k}{N} R_c \alpha \bar{\mu}_\phi \\ \bar{H} = \dfrac{1}{r}\,\mathrm{grad}\,(r\alpha) \times \bar{\mu}_\phi \end{cases} \tag{1}$$

k is the wavenumber in the dielectric and $R_c = \sqrt{\mu_0/\epsilon_0} = 120\pi$ the free-space impedance. r, ϕ, z are cylindrical coordinates with the z-axis along the symmetry axis of the resonator, a meridian cross section of which appears in Fig. 1. By substituting (1) in Maxwell's equations, we find that

$$\begin{cases} \mathscr{L}\alpha + k^2\alpha = 0, & \text{in } S \\ \mathscr{L}\alpha + \dfrac{k^2}{N^2}\alpha = 0, & \text{in } S' \text{ and } S'' \end{cases} \tag{2}$$

where S is the cross section of the inner volume of the resonator and S' and S'' of the outer volume. The differential operator $\mathscr{L}$ is

$$\mathscr{L}\alpha = \frac{\partial^2\alpha}{\partial r^2} + \frac{1}{r}\frac{\partial \alpha}{\partial r} + \frac{\partial^2\alpha}{\partial z^2} - \frac{1}{r^2}\alpha. \tag{3}$$

The functions α and $\partial\alpha/\partial n$ are continuous on C (the interface between resonator and vacuum), while α is zero on the z-axis and regular at infinity. For a dipole mode, α is symmetric about the $z = 0$ plane, while for a quadrupole mode it is antisymmetric. To apply the perturbational

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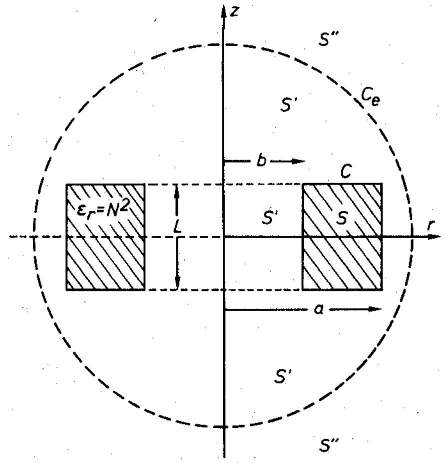

Fig. 1. Coordinates and dimensions of a ring resonator in free space—merdian cross section.

approach, the relevant power expansions are [15]

$$\begin{cases} \alpha = \alpha_0 + \dfrac{1}{N^2}\alpha_2 + \dfrac{1}{N^4}\alpha_4 + \cdots \\ k^2 = k_0^2 + \dfrac{1}{N^2}(k^2)_2 + \dfrac{1}{N^4}(k^2)_4 + \cdots \end{cases} \tag{4}$$

where we shall allow $(k^2)_2$ and $(k^2)_4$ to be positive or negative. It can be shown that only even powers in $1/N$ are present in (4) [15]. Inserting these values in (2) leads to the known equation for α_0 [11], and the following equation for α_2:

$$\begin{cases} \mathscr{L}\alpha_2 + k_0^2\alpha_2 = -(k^2)_2\alpha_0 & \text{in } S \\ \mathscr{L}\alpha_2 = -k_0^2\alpha_0 & \text{in } S' \text{ and } S''. \end{cases} \tag{5}$$

This equation determines α_2 to within an arbitrary multiple of α_0. The resulting indeterminacy is classically removed by means of an orthogonality condition [16] which is, in the present case [9]

$$\iiint_{V_i+V_o} \overline{H}_m \cdot \overline{H}_p \, dV = 0 \tag{6}$$

where $\overline{H}_m$ and $\overline{H}_p$ are the magnetic fields of two different modes and V_i and V_o are the inner and outer volume of the resonator, respectively. Applied to α_0 and α_2, (6) reduces to

$$\iint_S \alpha_0\alpha_2 r \, dS = 0. \tag{7}$$

B. Resonant Wavenumber

The fundamental relationship for the determination of the higher perturbational orders is

$$\iint_S [f\mathscr{L}g - g\mathscr{L}f] r \, dS = \int_C \left[f\frac{\partial g}{\partial n} - g\frac{\partial f}{\partial n} \right] r \, dc \tag{8}$$

with n the outward normal. Suitable use of (8) leads to

$$(k^2)_2 = -k_0^2 \frac{\iint_{S'+S''} \alpha_0^2 r \, dS}{\iint_S \alpha_0^2 r \, dS}. \tag{9}$$

The expression for $(k^2)_4$ requires knowledge of α_0 and α_2, a normal requirement in an iterative process, and a particular solution of α_4 in the outer region, which is easily derived from α_0 and α_2 without knowledge of $(k^2)_4$. Thus

$$\begin{aligned} (k^2)_4 = -\frac{1}{\iint_S \alpha_0^2 r \, dS} & \cdot \Bigg\{ (k^2)_2 \left[\iint_S \alpha_0\alpha_2 r \, dS + \iint_{S'+S''} \alpha_0^2 r \, dS \right] \\ & + k_0^2 \lim_{R_\infty \to \infty} \Bigg[\iint_{S'+S''} \alpha_0\alpha_2 r \, dS - \frac{1}{k_0^2} \int_{C_\infty} \\ & \cdot \left(\alpha_4 \frac{\partial \alpha_0}{\partial n} - \alpha_0 \frac{\partial \alpha_4}{\partial n} \right) r \, dc \Bigg] \Bigg\} \end{aligned} \tag{10}$$

where C_∞ is a circle of large radius R_∞ in the meridian plane. In (10), we have made no use of the normalization (7), as (10) is insensitive to the undeterminacy of α_2.

C. Quality Factor Due to Radiation Losses

To find Q_r, the quality factor due to radiation losses, we use the general relationship [9]

$$Q_r = \frac{kc}{N} \frac{\mathscr{E}}{\mathscr{P}_r} = Q_0 \left(1 + \frac{1}{N^2} \frac{Q_2}{Q_0} + \cdots \right) \tag{11}$$

where $\mathscr{P}_r$ is the radiated power, and $\mathscr{E}$ the total stored field energy. A detailed evaluation, shown in Appendix I, leads to the following Q for the dipole mode:

$$\begin{aligned} Q_r = \frac{12N^3}{k_0^5} \frac{\iint_S \alpha_0^2 r \, dS}{\left[\iint_S \alpha_0 r^2 \, dS \right]^2} & \Bigg\{ 1 + \frac{1}{N^2} \Bigg[2 + \frac{7}{2} \frac{\iint_{S'+S''} \alpha_0^2 r \, dS}{\iint_S \alpha_0^2 r \, dS} \\ & + 2 \frac{\iint_S \alpha_0\alpha_2 r \, dS}{\iint_S \alpha_0^2 r \, dS} - 2 \frac{\iint_S \alpha_2 r^2 \, dS}{\iint_S \alpha_0 r^2 \, dS} \\ & + \frac{k_0^2}{5} \frac{\iint_S \alpha_0 (r^2 + z^2) r^2 \, dS}{\iint_S \alpha_0 r^2 \, dS} \Bigg] \Bigg\}. \end{aligned} \tag{12}$$

For the magnetic quadrupole mode, a similar expression obtains, which is now proportional with N^5. As for $(k^2)_4$, use of the normalization condition (7) is not necessary for a correct evaluation of the correction term in (12).

D. Numerical Implementation

The regions S and S' are divided into triangular elements in which higher order polynomials are used. In the exterior region S'', the field is represented by a finite sum of static spherical harmonics with unknown coefficients. On C_e, we enforce the continuity of the finite-element

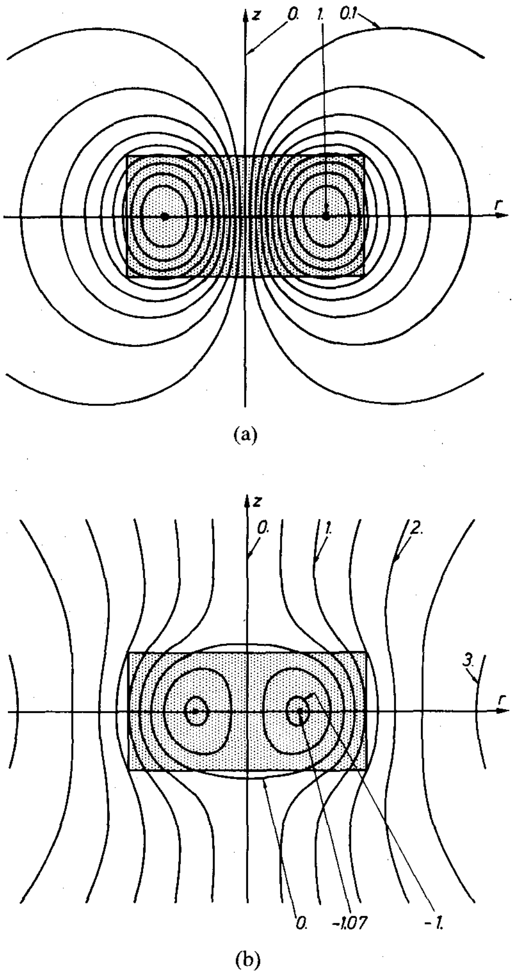

Fig. 2. Zero- and second-order field of a pillbox with aspect ratio $L/2a = 0.5$ (dipole mode). (a) Zero-order field α_0. (b) Second-order field α_2.

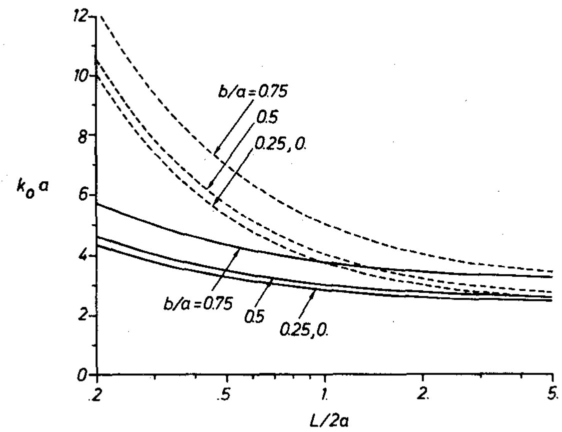

Fig. 3. Zero-order of the resonant wavenumber $k_0 a$ versus aspect ratio $L/2a$ and $b/a = 0$, 1/4, 1/2, and 3/4. Full line —— dipole mode. Dashed line ---- quadrupole mode.

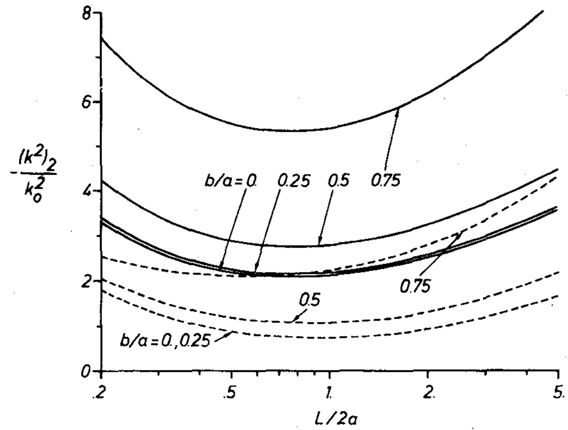

Fig. 4. First correction of the resonant wavenumber $-(k^2)_2/k_0^2$ versus aspect ratio $L/2a$ and $b/a = 0$, 1/4, 1/2, and 3/4. Full line —— dipole mode. Dashed line ---- quadrupole mode.

functions and the series (Fig. 1). This method is very reliable and yields excellent field values both inside and outside the resonator, which are quite independent of the position of the outer contour C_e [17]. Checked against the known analytical solution of the sphere, the accuracy on the resonant wavenumber and the various integrals of α is found better than 0.1 percent. In practice, we have used about 70 third-order finite elements (346 vertices) and 9 terms in the expansion in the exterior region S''.

III. Numerical Results

We have applied our analysis to the ring resonator of Fig. 1, the dipole mode of which has already been investigated in the zero order [11]. We have also considered, for purposes of verification, the spherical resonator for which analytical solutions are available, both for arbitrary N and the present asymptotic series [15], [18].

A. Fields

We have selected data for a pillbox of aspect ratio $L/2a = 1/2$ and radiating as a magnetic dipole. Fig. 2(a) shows the lines of constant α_0, normalized to a maximum $\alpha_0 = 1$, and separated by steps of 0.1. Fig. 2(b) displays the corresponding values of α_2. The large distance behavior of these functions is of interest. For α_0, it is $\sin\theta/R^2$ and $\sin\theta$ for α_2. We note that, in the case of the quadrupole mode, this behavior is $\sin\theta\cos\theta/R^3$ for α_0 and $\sin\theta\cos\theta/R$ for α_2.

B. Resonant Wavenumber

Plots of $k_0 a$, $-(k^2)_2/k_0^2$, and $(k^2)_4/k_0^2$ are given in Figs. 3–5 for various geometrical ratios. The corresponding numerical data for the dipole mode can be found in Table I. In the figures, the full lines represent the results for the dipole mode, while the dashed lines refer to the quadrupole mode. In Fig. 5, $(k^2)_4/k_0^2$ is positive for the dipole mode and negative for the quadrupole mode. The data are of great interest for practical applications. An idea of the accuracy they provide is obtained by applying the perturbational method to the sphere [19]. For the sphere, using one term, two terms, or three terms in the expansion (4) of ka, respectively, gives a relative error of about 1, 0.1, or 0.01 percent at $\epsilon_r = 100$; 2.1, 0.54, or 0.17 percent at $\epsilon_r = 39$; and 3, 1.2, or 0.58 percent at $\epsilon_r = 25$ for the dipole mode. For the quadrupole mode, we find, respectively, (0.36, 0.02, 0.003 percent), (1, 0.1, 0.05 percent), and

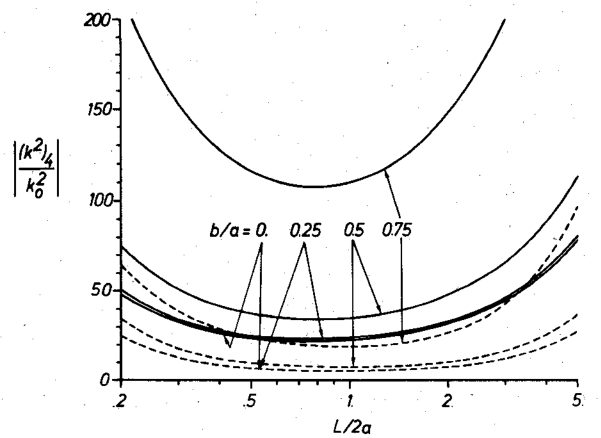

Fig. 5. Second correction of the resonant wavenumber $|(k^2)_4/k_0^2|$ versus aspect ratio $L/2a$ and $b/a = 0, 1/4, 1/2,$ and $3/4$. Full line —— dipole mode. Dashed line ---- quadrupole mode.

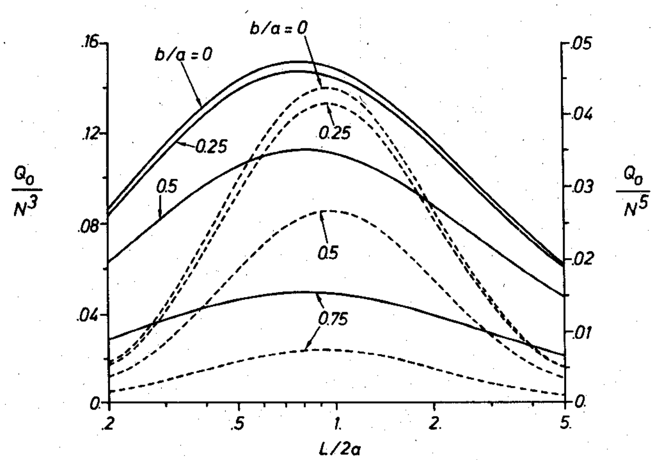

Fig. 6. Zero-order of the Q-factor Q_0 versus aspect ratio $L/2a$ and $b/a = 0, 1/4, 1/2,$ and $3/4$. Full line —— Q_0/N^3 (dipole mode). Dashed line ---- Q_0/N^5 (quadrupole mode).

TABLE I

RESONANT WAVENUMBER VERSUS ASPECT RATIO $L/2a$ FOR $b/a = 0, 1/4, 1/2, 3/4$ (DIPOLE MODE). THE FIRST PART IS RELEVANT TO $k_0 a$, THE SECOND ONE TO $-(k^2)_2/k_0^2$, THE THIRD ONE TO $(k^2)_4/k_0^2$.

b/a \ L/2a	1/5	1/3	1/2	2/3	1	3/2	2	3	5
.0	4.338	3.652	3.259	3.046	2.822	2.667	2.590	2.515	2.461
.25	4.363	3.673	3.277	3.063	2.837	2.681	2.603	2.527	2.472
.50	4.635	3.906	3.485	3.256	3.013	2.844	2.759	2.677	2.616
.75	5.731	4.857	4.346	4.063	3.759	3.545	3.436	3.329	3.249
.0	3.309	2.502	2.189	2.098	2.122	2.300	2.507	2.897	3.547
.25	3.404	2.579	2.253	2.158	2.180	2.357	2.570	2.966	3.623
.50	4.237	3.265	2.879	2.762	2.779	2.980	3.224	3.690	4.461
.75	7.442	6.051	5.494	5.334	5.394	5.748	6.169	6.965	8.286
.0	48.14	29.92	23.91	22.29	22.81	26.62	31.90	44.64	77.87
.25	50.63	31.38	25.03	23.28	23.77	27.69	33.16	46.33	80.39
.50	75.09	46.89	37.36	34.60	35.00	40.27	47.74	65.98	112.5
.75	216.3	142.2	116.1	108.5	109.9	125.2	147.1	199.4	331.4

(1.6, 0.21, 0.17 percent) for the same values of ϵ_r. A remark of general interest—the correction term $(k^2)_2/k_0^2$ is smaller for the quadrupole mode than for the dipole mode. This is because the zero-order fields are more strongly concentrated in the dielectric region for the quadrupole mode than for its dipole counterpart.

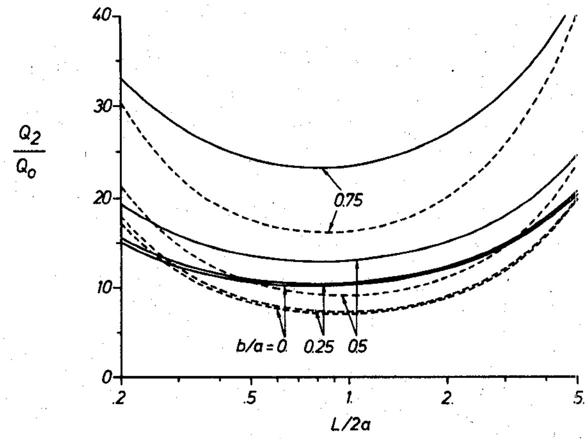

Fig. 7. Correction of the Q-factor Q_2/Q_0 versus aspect ratio $L/2a$ and $b/a = 0, 1/4, 1/2,$ and $3/4$. Full line —— dipole mode. Dashed line ---- quadrupole mode.

TABLE II

RADIATION QUALITY-FACTOR VERSUS ASPECT RATIO $L/2a$ FOR $b/a = 0, 1/4, 1/2, 3/4$ (DIPOLE MODE). THE UPPER PART IS RELEVANT TO THE ZERO ORDER Q_0/N^3, THE LOWER ONE TO THE FIRST CORRECTION Q_2/Q_0.

b/a \ L/2a	1/5	1/3	1/2	2/3	1	3/2	2	3	5
.0	.08636	.1219	.1434	.1508	.1482	.1321	.1157	.09061	.06167
.25	.08351	.1181	.1391	.1465	.1441	.1287	.1129	.08843	.06036
.50	.06306	.08907	.1053	.1114	.1104	.09935	.08756	.06900	.04742
.75	.02890	.03978	.04649	.04907	.04875	.04415	.03914	.03109	.02155
.0	15.03	11.82	10.56	10.18	10.28	11.10	12.16	14.58	20.34
.25	15.48	12.17	10.84	10.44	10.53	11.34	12.43	14.88	20.68
.50	19.24	15.16	13.50	12.97	12.99	13.89	15.13	17.95	24.57
.75	33.07	26.82	24.20	23.36	23.43	24.92	26.96	31.56	42.17

C. *Quality Factor Due to Radiation Losses*

The relevant information is contained in Figs. 6 and 7 and Table II. An idea of the accuracy is provided by checking versus the sphere [19]. Including one or two terms in Q_r yields accuracies of 8.8 or 0.25 percent at $\epsilon_r = 100$; 19 or 1.3 percent at $\epsilon_r = 39$; and 26 or 2.6 percent at $\epsilon_r = 25$ for the dipole mode. For the quadrupole mode, we find (6.7 and 0.43 percent), (17 and 2.4 percent), and (25 and 5.1 percent), respectively. Note from Fig. 7 that the correction term Q_2/Q_0 can become larger for the quadrupole mode than for the dipole mode. This is in contrast with the first correction $(k^2)_2/k_0^2$, as Q_2/Q_0 also involves the first correction field α_2.

IV. CONCLUSIONS

The leading term in the asymptotic theory is sufficient for the study of resonators with an ϵ_r of about 100. Present-day materials tend to have lower ϵ_r's, of the order of 40, and the results of the "leading term" approximation might not be sufficiently accurate. Tsuji *et al.* [12] have developed a method which yields excellent results for arbitrary ϵ_r, but requires a separate solution for each ϵ_r under consideration. These authors, recognizing the advantage of the asymptotic procedure, mention that the latter should ideally be extended to higher order terms in $1/N$, but that such an extension would be very complicated. We believe that the present paper shows that these complications are minor. We have, in fact, obtained the next correction term for the fields, the resonant wavenumber, and the quality factor. These terms are of the order $1/N^2$. The next term of the resonant wavenumber, which is of the order $1/N^4$, has also been evaluated. Extensive additional data on these and other parameters, such as the dipole or quadrupole moment, higher order multipoles, far-field, near-field, etc., $\cdots$, are available for the ring resonator, but have not been included because of a lack of space [15].

It is extremely difficult to check the convergence of the asymptotic series with $1/\epsilon_r$ as a small parameter. Our formulas, when applied to a spherical resonator, show that use of the available corrections yields good values at $\epsilon_r = 39$ and still tolerable results at $\epsilon_r = 25$. Analogous accuracies for the ring resonator can be expected for an aspect ratio $L/2a$ in the range $(1/2, 3/2)$ and an inner hole in the range $(b/a < 0.25)$. Outside this range, no precise indications on the accuracy are available, but larger errors may be expected. We have also compared our results with recent ones presented by Tsuji *et al.* for a pillbox [12]. For ϵ_r higher than 25, the difference is less than 0.1 percent for the resonant wavenumber and less than 1 percent for the Q-factor. Hence, for this range of ϵ_r, we may expect a good convergence of the asymptotic series. Due to the asmptotic nature of the expansions, the results do diverge for low values of ϵ_r, such as 10 or less.

It is perhaps useful to reemphasize that the main merit of the asymptotic method is to avoid repetitive calculations. Its validity for arbitrary (but sufficiently high) ϵ_r allows one to quickly investigate, without additional computations, the effect of variations of ϵ_r on the properties of the resonator. These variations may be caused by factors such as the temperature or the fluctuating nature of the fabrication process. The relative shift of the resonant frequency due to a variation in ϵ_r, for example, is easily derived from (4) to be

$$\frac{\Delta f_{\text{res}}}{f_{\text{res}}} \simeq -\frac{1}{2}\left(1+\frac{(k^2)_2}{\epsilon_r k_0^2}+\cdots\right)\frac{\Delta\epsilon_r}{\epsilon_r}. \tag{13}$$

Appendix
Evaluation of the Q-Factor

The radiated fields are generated by the polarization current

$$\bar{J}=\left(1-\frac{1}{N^2}\right)k^2\alpha\bar{u}_\phi \tag{14}$$

which differs from zero only in V_i, the inner volume of the resonator. With (14), the magnetic vector potential, respectively, everywhere and in the far-field, is [20]

$$\bar{A}=\frac{\mu_0}{4\pi}\iiint_{V_i}\bar{J}(\bar{r}')\frac{e^{-j\frac{k}{N}|\bar{r}-\bar{r}'|}}{|\bar{r}-\bar{r}'|}dV' \approx \frac{e^{-j\frac{k}{N}R}}{R}\bar{N} \tag{15}$$

where $\bar{N}$ is independent of R, the distance from a point in the far-field to the center of the resonator. The power radiated by (15) is

$$\mathscr{P}_{\text{rad}}=\frac{1}{2}\frac{k^2}{N^2}R_c\iint_\Omega\left|\frac{\bar{N}}{\mu_0}\times\bar{u}\right|^2 d\Omega \tag{16}$$

where the integral extends over all directions $\bar{u}$. The method proceeds now by inserting the series (4) into (14) and retaining only 2 successive terms in the series in $1/N^2$ in all the calculations.

The total stored field energy $\mathscr{E}$ is the sum of the electric and magnetic energy

$$\mathscr{E}=\frac{\epsilon_0}{4}\left[\iiint_{V_i}\epsilon_r|\bar{E}|^2 dV+\iiint_{V_o}|\bar{E}|^2 dV\right]+\frac{\mu_0}{4}\iiint_{V_i+V_o}|\bar{H}|^2 dV \tag{17}$$

where V_o is the outer volume of the resonator. It can be proven that, at resonance, the electric and magnetic energy are equal. Substituting the field components (1) into (17) we find, up to the first order in $1/N^2$

$$\mathscr{E}=\mu_0\pi k_0^2\left(\iint_S\alpha_0^2 r\,dS\right)\left[1+\frac{2}{N^2}\frac{\iint_S\alpha_0\alpha_2 r\,dS}{\iint_S\alpha_0^2 r\,dS}+\cdots\right]. \tag{18}$$

With the normalization (7) the total stored field energy becomes, up to the first order, independent of N. Combining (16) and (17) in (11) gives the final result (12).

Acknowledgment

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Theory and Numerical Simulation of a TE_{111} Gyroresonant Accelerator

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***Abstract*—The production of spiral relativistic electron beams in a TE_{111} gyroresonant accelerator cavity for injection into a compact high-harmonic gyrotron is studied. Parametric studies are performed to determine the effects of variations in the background magnetic field amplitude, the RF amplitude in the cavity, and the initial beam voltage on the output beam. The effects of velocity spread and a finite radial extent of the input beam are also discussed. Power curves for obtaining optimum operating regimes for the TE_{111} accelerator are provided.**

I. Introduction

GYROTRONS have successfully generated electromagnetic radiation in the millimeter and submillimeter wavelength ranges via the electron-cyclotron maser instability [1]–[3]. Electromagnetic radiation is produced through the interaction of a relativistic electron beam gyrating about an external magnetic field and an excited cavity mode which grows at the expense of the rotational energy of the beam. The production of relativistic electron beams, with most of its kinetic energy in the form of rotational energy, therefore, plays a crucial role in the development of gyrotron devices.

In this paper, we discuss an "injection system" capable of producing such a beam for use in a compact high-harmonic gyrotron [4], [5]. This system, which consists of a conventional electron gun and a resonant "accelerator" cavity, has two advantages over other systems currently in use: 1) the system is technically simple, and 2) it operates at a relatively low voltage. It also differs from relativistic electron-beam sources used in conventional gyrotrons in that it produces a beam whose Larmor orbits encircle the cavity axis. Conventional gyrotrons operate at the first or second harmonic with a relatively high magnetic field, and with Larmor orbits of the beam particles which are much smaller than the cavity radius. In contrast, the compact high-harmonic gyrotron operates with a low magnetic field and with Larmor orbits of the beam particles which are comparable to the cavity radius, and which encircle the cavity axis. This may permit the construction of gyrotrons based on permanent magnet technology such as samarium–cobalt.

Typical particle orbits in a conventional gyrotron are shown in Fig. 1(a) and the axis-encircling orbits of the compact high-harmonic gyrotron are shown in Fig. 1(b). The remainder of the introduction consists of a brief description of the current beam production methods with emphasis on their advantages and disadvantages. Finally, the novel system, which is the object of this paper, is discussed.

Most electron guns produce a monoenergetic, unidirectional beam. The obvious way to produce a beam with a large fraction of rotational energy would be to inject the beam at an angle to the background magnetic field [6]. This angle would determine the partition of energy in the beam between the perpendicular and parallel components.

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